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# Rapid Verification of Engine Rotor and Case Flexibilities by a Modal Comparison Algorithm

Ronald A. Marmol* and Joe T. Akin†

*Pratt & Whitney Aircraft Division, United Aircraft Corporation, West Palm Beach, Fla.*

**A computation technique has been developed at Pratt & Whitney Aircraft (P&WA) in which localized dynamic flexibilities in an assembled rotor or case can be rapidly determined from experimental mode shape and frequency data. A dynamic mathematical model of the structure is developed with empirical flexibility terms assigned to mechanical joints such as flanges, splines, couplings, etc. The vibratory response of the structure is measured in laboratory tests and compared with calculated values. Agreement between calculated and experimental mode shapes and frequencies is obtained by a computerized random search technique, which determines the flexibility terms that produce the best match between experimental data and calculated values for all of the vibration modes compared. The technique was developed for rotor critical speed applications, but it may be applied to any simple or complex beam type structure.**

## I. Introduction

THE emphasis on a lightweight, high-performance design for advanced turbojet engines has required the designer to use mechanical joints (flanges, splines, couplings, etc.), which are significantly different from earlier designs. These new joints influence the dynamic response of the structure and, consequently, affect the engine's sensitivity to unbalance, stall loading, and maneuver deflections. To verify the adequacy of the joints before an engine test, the designer must conduct design verification tests to corroborate his predicted flexibilities. These test results must be interpreted and the impact on design performance evaluated in time to make design modifications if necessary. The sophisticated mathematical models, which are required to accurately compute the engine dynamic response, should therefore be arranged so that the data from design verification tests can be readily incorporated and the data's impact quickly evaluated.

The subject of this paper is the rapid incorporation in an existing math model of joint flexibilities derived from structural test data. A computational technique has been developed in which flexibilities that occur at mechanical joints in an assembled rotor or case are determined from laboratory mode shape tests. In this technique, empirical flexibility terms assigned to mechanical joints (flanges, splines, couplings) are used in the initial calculation to compare with experimental data. Flexibilities are then varied, if necessary, by computerized calculation until the best match is obtained between calculated and experimental mode shapes. The result is a math model, with joint flexibilities defined, that accurately describes the dynamic stiffness and mass distribution of the assembled rotor or case.

The use of this technique has considerably reduced the time necessary to adjust the math model to account for local flexibilities, which may not have been accurately predicted during the design phase. Perhaps more importantly, it has shown that a rather subtle change in the overall stiffness of an assembled rotor due to localized joint flexibilities can cause a significant increase in the vibration response. This is illustrated by an example problem on a high-pressure compressor rotor.

## II. Technique

### A. Methodology

#### 1) *Math model development*

A large number of vibratory systems in engineering are described by what is commonly known as a lumped mass mathematical model of the true physical system. This model can be used to determine the natural frequencies, mode shapes, forced response, etc., of the physical system and will accurately duplicate the system if local joint flexibilities are known.

To develop the math model, the structure is represented by a series of stations connected by massless beam sections of cylindrical, conical, or plate elements. The stations are chosen to coincide with concentrated masses, span ends, and locations of geometry change. Each station is assigned a mass and a mass moment of inertia, which includes the beam inertias as well as those for concentrated masses such as disks or gears. From these properties and the material characteristics, bending and shear flexi-

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*Senior Analytical Design Engineer, Florida Research and Development Center.

†Assistant Design Project Engineer, Florida Research and Development Center.

bilities are calculated and used in a transfer matrix multiplication similar to the Prohl method[3] to determine the natural frequencies of beam-like structures. Because the flexibilities of the mechanical joints are determined from empirical data, their values are direct input items to the math model.

*2) Acquisition of experimental data*

Experimental mode shape and natural frequency data are obtained in laboratory tests using mechanical impedance equipment for correlation with results from the math model. The structure to be tested is assembled and supported in the horizontal position on supports with negligible stiffness so that the free-free bending modes of the structure are not affected. Accelerometers are located axially along the structure at close intervals so that accurate mode shapes can be obtained.

Mechanical impedance data (force/velocity) are then recorded in the usual way for several pickups.[1] These data are used to verify the system instrumentation accuracy and locate the resonant frequencies. Instrumentation accuracy is checked by Betti's reciprocal theory[2] which requires that the transfer impedance of a structure measured at a Point "A" with the excitation applied at Point "B" will be equal to the transfer impedance measured at Point "B" with the excitation applied at Point "A." Natural frequencies are detected by tuning the exciting frequency until the point impedance has a 90° phase lag, which is characteristic of a resonance. Mode shape data are then taken by applying a constant driving force at the resonant frequencies of interest and recording acceleration amplitudes and phase angles at each accelerometer location.

*3) Determination of joint flexibility*

Once the experimental data are obtained, mechanical joint flexibilities can be determined through a mode shape and resonant frequency matching computer technique called FLEXID (Fig. 1), which was developed at Pratt & Whitney Aircraft. Basically, FLEXID is a coupling of the Prohl type method for mode shape and resonant frequency prediction and a statistically controlled random search routine referred to as a stochastic search. The search routine varies the joint flexibilities in the math model until the best match is obtained with the experimental data, which are input to FLEXID.

The best match is defined as the one that has the smallest differences between the analytical and experimental resonant frequencies and mode shapes. Mathematically, these differences are expressed in the form of a quadratic equation referred to as a cost function *(F)*. Following the approach of Ref. 5, the function is the sum of the weighted squares of the differences between experimental and theoretical resonant frequencies and relative deflections:

$$F = \sum_{i=1}^{n} \{W_{i_f}(\omega_{i_e} - \omega_{i_t})^2 + W_{i_x} \sum_{j=1}^{n} (X_{ij_e} - X_{ij_t})^2\} \quad (1)$$

The frequencies and deflections are represented by $\omega$ and $X$, respectively. Subscript notations $i$ and $j$ denote the resonance number and corresponding relative deflection position, respectively; subscripts $e$ and $t$ represent the experimental and theoretical parameters. The weighting factors of frequency and mode shape, $W_{i(f)}$ and $W_{i(x)}$, respectively, assign relative importance to each structural mode, i.e., data quality or physical importance of higher frequency modes may dictate that data from those modes be considered less important than the lower modes.

To determine the minimum of the cost function, a stochastic search approach[4] was incorporated into FLEXID to preclude the convergence problems encountered with

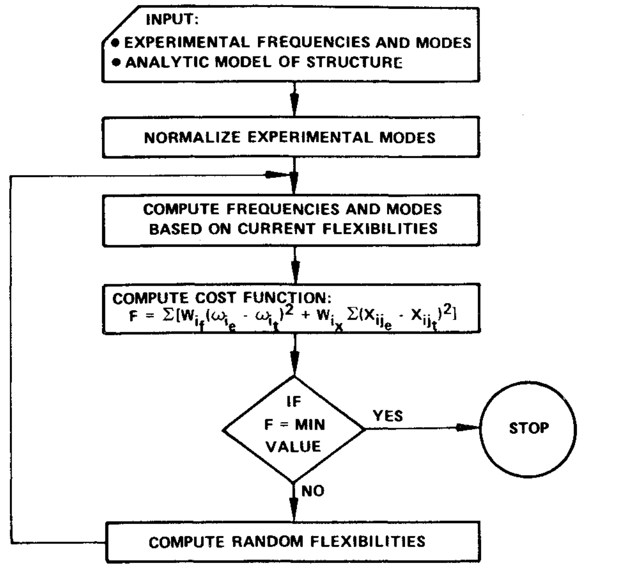


**Fig. 1 FLEXID—mechanical joint flexibility identification analysis.**

the gradient method used in Ref. 5. At present, FLEXID is capable of solving for ten independent flexibilities simultaneously and can treat a practical rotor or case problem in less than 3 min on an IBM 370 model 168 computer.

In addition to its main function as described, FLEXID also performs other operations that are very useful and time saving. It accepts the raw test data (frequency, *g* levels, and phase angle), converts it into normal mode response, and then scales the theoretical response for direct comparison purposes. A direct output of FLEXID is a graphic comparison of theoretical and experimental results as shown in an example problem. A brief explanation of the techniques used in these operations is given here.

Theoretically, a structure's displacement vectors, at a pure resonance, are 90° out of phase with the excitation source. In practice, however, a scatter in phase angle is observed at the structure's resonant peaks. This scatter is due to instrumentation error and the response from resonances other than the normal mode resonance investigated. To obtain the experimental normal mode response and eliminate other mode contributions and instrumentation errors, a least squares deviation procedure is used. Since the normal mode response must lie in a single plane, it is determined by a least squares deviation from

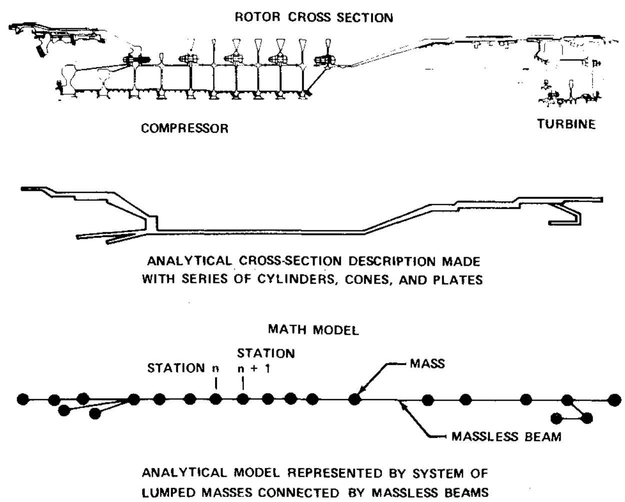


**Fig. 2 Rotor model development.**

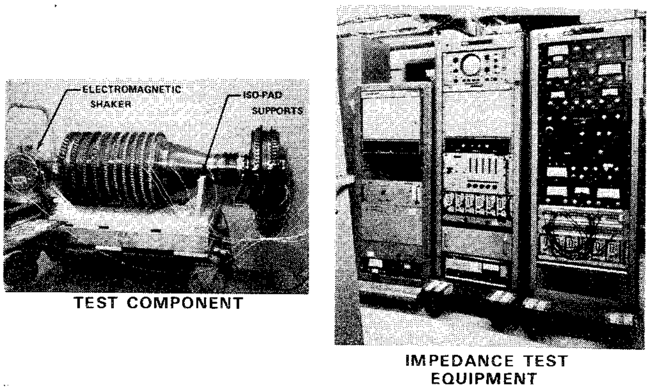


**Fig. 3 Rotor and test equipment.**

the normal mode plane through the experimental data. The least squares deviation (G) in polar coordinates is defined as:

$$G = \Sigma\{X_i \sin(\theta_i - \phi_i)\}^2 \tag{2}$$

where $X_i$ and $\theta_i$ are deflection and phase of experimental data for that mode and $\phi$ is the angle defining the plane of normal mode.

To determine the minimum deviation:

$$\frac{\partial G}{\partial \phi} = 0 = \sin 2\phi \Sigma X_i^2 (\cos^2\theta_i - \sin^2\theta_i) - 2\cos 2\phi \Sigma X_i^2 \sin\theta_i \cos\theta_i \tag{3}$$

To solve for the normal plane:

$$\tan 2\phi = \frac{2\Sigma X_i^2 \sin\theta_i \cos\theta_i}{\Sigma X_i^2 (\cos^2\theta_i - \sin^2\theta_i)} \tag{4}$$

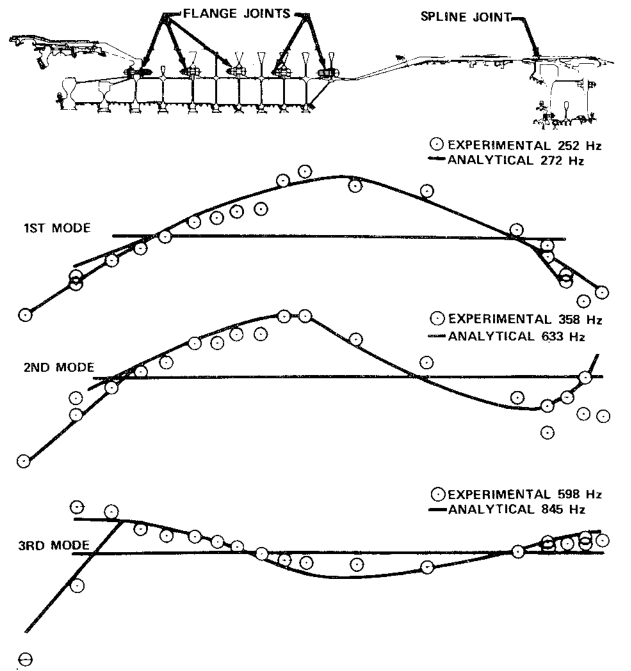


**Fig. 4 Test results show that the rotor joint flexibilities are higher than estimated.**

$$\phi = 1/2 \tan^{-1}\left[\frac{2\Sigma X_i^2 \sin\theta_i \cos\theta_i}{\Sigma X_i^2 (\cos^2\theta_i - \sin^2\theta_i)}\right] \tag{5}$$

The experimental data are now projected onto this normal mode plane by:

$$X_i' = X_i \cos(\phi - \theta_i) \tag{6}$$

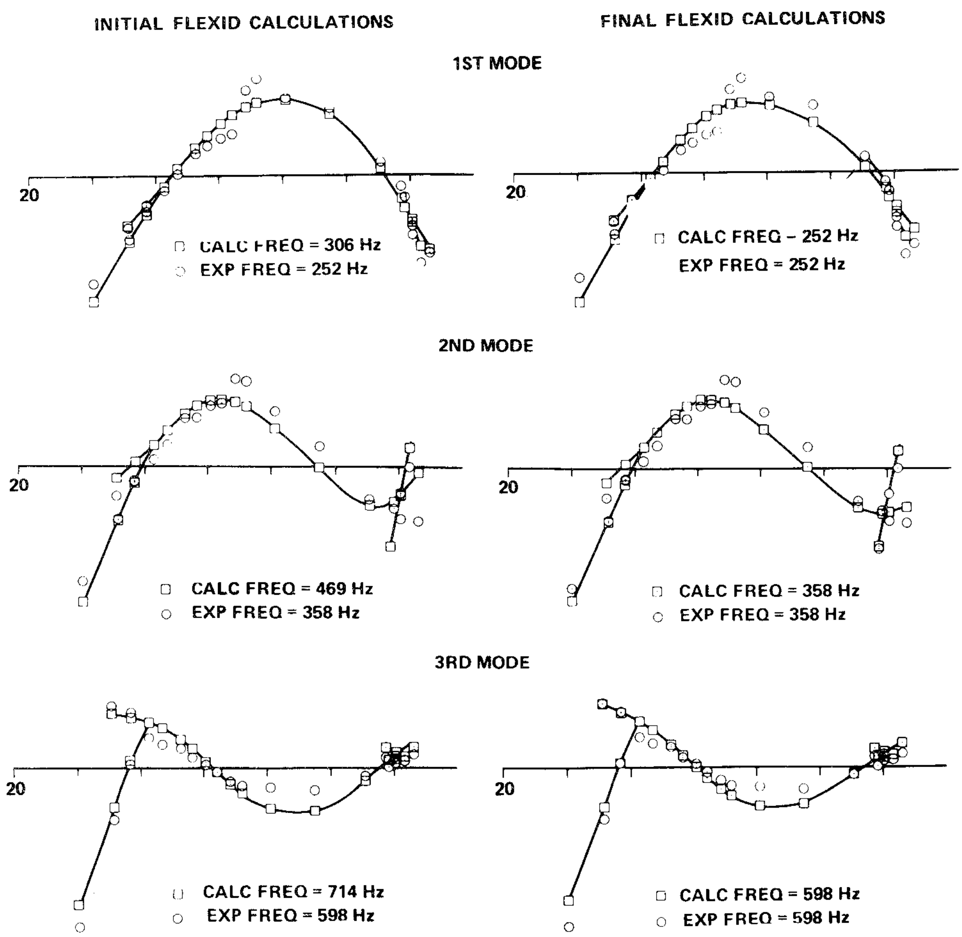


**Fig. 5 Joint flexibilities increased until math model exactly matches experimental results.**

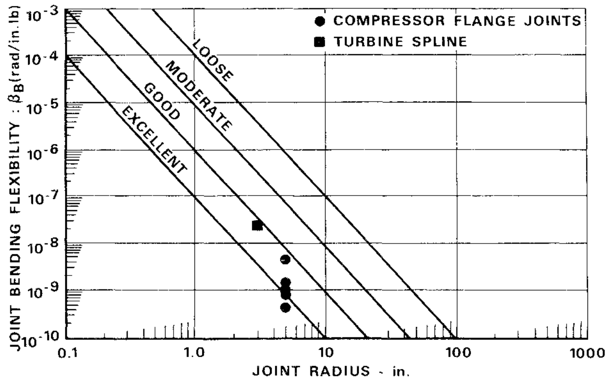


Fig. 6 NASA joint flexibility rating.

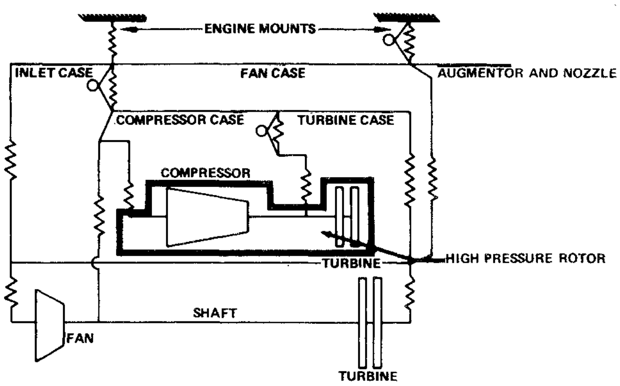


Fig. 7 Engine dynamic model.

After the experimental normal mode deflections are determined, a scale factor is applied to theoretical deflections; this reduces the differences between the theoretical and experimental values to a minimum and permits direct comparisons. The scale factor $S$ is determined by minimizing the least square difference $(H)$ between the experimental $(X_e)$ and theoretical $(X_t)$ displacement data:

$$H = \Sigma(X_e - SX_t)^2 \qquad (7)$$

To minimize with respect to $S$:

$$\partial H/\partial S = \Sigma(-2X_eX_t + 2SX_t^2) = 0 \qquad (8)$$

The scale factor is then:

$$S = (\Sigma X_eX_t)/(\Sigma X_t^2) \qquad (9)$$

### B. Example of the Technique Applied to Verify Joint Flexibilities in a High-Pressure Compressor Rotor

The computer program FLEXID was recently used at P&WA's Florida Research and Development Center to verify the local flexibilities used in the structural math model on the high-pressure rotor of a turbofan engine. Because the performance of this engine is highly sensitive to blade tip clearance, an accurate model is required to evaluate the rotor dynamic response. Furthermore, because minimum weight was an important design goal, scalloped minimum thickness joints were used; this deviated significantly from previous designs.

A cross section of the test rotor and a schematic of the math model used are shown in Fig. 2. Local flexibilities based on previous empirical data were assigned to flange joints in the compressor and the turbine spline; the rotor free-free natural frequencies and mode shapes were calculated. The rotor analytical model was taken from the engine model and modified to represent rotor test conditions. This was done through the modification of the rotor modulus of elasticity to its room temperature value and subtracting rotor support springrate and gyroscopic effects to simulate test conditions. In addition, the blades were removed from the test rotor to preclude any problems due to loose attachments; the model was modified accordingly.

The natural frequencies of the assembled rotor were determined by exciting the rotor with an impedance head attached to a mechanical shaker through a 70 to 5000 Hz frequency range (Fig. 3). At the natural frequencies of interest, amplitudes and phase angles at each accelerometer were then recorded so that normal mode shapes could be determined by the method described in Sec. IIA. A comparison of the analytical and experimental results obtained for the first three free-free bending modes is shown in Fig. 4. All three modes were lower than predicted, however, by only 7.5% in the first mode. To correct the analytical model, the computer technique FLEXID was applied and flexibilities at the flanges and spline were increased until exact agreement in natural frequencies was obtained as shown in Fig. 5.

A comparison of the resulting joint flexibilities with a NASA rating (Fig. 6) shows that they are in the good to excellent range meaning "fairly rigid" joints.[6] The scatter in the flange flexibilities is caused by either subtle differences in design details or a limitation in individual joint accuracy due to the simultaneous solution of several unknowns. In the latter case, additional experimental modes could have been obtained, which would have increased the accuracy of the flexibilities, but this was not the primary objective of the test. The spline coupling had a larger effect on overall rotor flexibility because it has a smaller diameter than any single flange joint.

To evaluate the impact of the experimental results on engine dynamics, the revised rotor model was incorporated into the engine dynamic model (Fig. 7). The effect on the engine high pressure rotor critical speeds and mode shapes is shown in Fig. 8. The 3rd mode, which is the rotor bending mode and the one most affected by the increased flexibilities, decreased by 2.5%; the first and second modes decreased less because they are basically rigid body modes (Fig. 8a). The critical speed results seem to show that the impact on engine unbalance response would be relatively minor because the rotor critical speeds were not significantly reduced; however, this is not necessarily

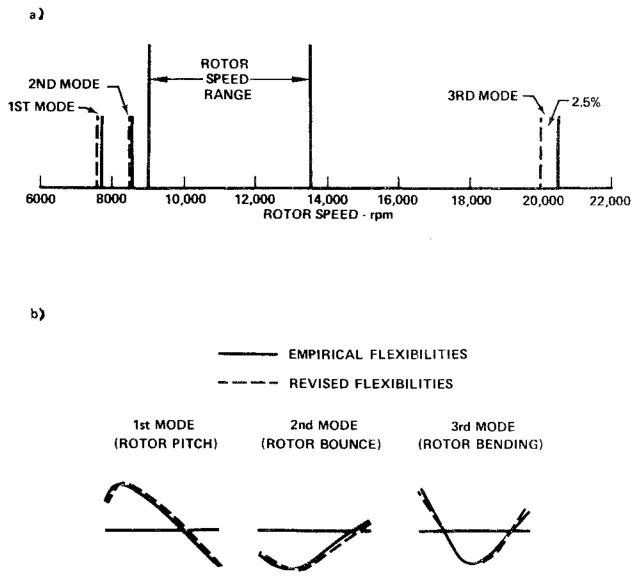


Fig. 8 The flexibilities determined by FLEXID permit a more accurate definition of the engine rotor modes: a) engine critical speeds; b) rotor mode shapes.

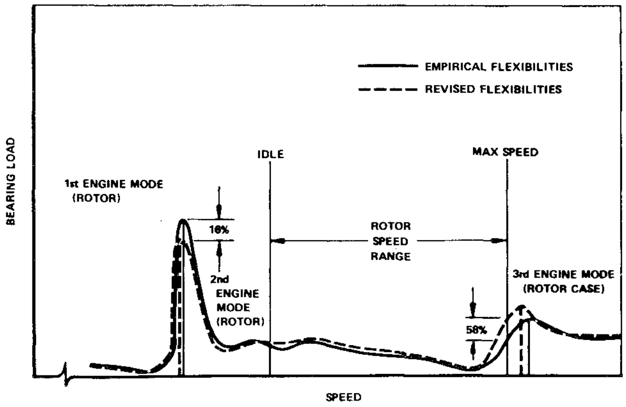


**Fig. 9 The increase in rotor joint flexibilities produces a 58% increase in the bearing load at maximum speed.**

true. In response studies, it was found that coupling effects, which significantly affected bearing load response, occurred when case critical frequencies approached natural rotor frequencies (Fig. 9). The maximum bearing load actually decreased by 16%; however, this load occurs at a transient condition and does not impact bearing life. At maximum rotor speed, which is a long time operating point, the bearing load is increased by 58% with the revised flexibilities. By increasing the rotor flexibilities to the values calculated by FLEXID, a small change in the rotor resonant frequency produces a significant change in bearing load.

### C. Other Applications

The example problem presented in this paper is for an assembled rotor where the flexibilities of several joints were computed with sufficient accuracy to permit correlation between experimental and theoretical analyses of overall rotor flexibility. FLEXID, which may be used for any structure that can be modeled as beam type elements, is a particularly efficient and accurate tool for determining the flexibility of a single joint when basic design data are desired. A research program to determine joint flexibilities by a combined analytical (finite element) experimental (impedance test/FLEXID) approach would significantly improve the capability of designing optimum mechanical joints (flanges, splines, coupling, etc.).

The Florida Research and Development Center of P&WA is currently using FLEXID to determine the flexibilities of flange joints used in the design of engine cases. These data and the flange data from the rotor test will be used to substantiate a finite element theoretical approach for predicting flange flexibility.

## IV. Conclusions

The computerized technique presented here has considerably reduced the time required to evaluate the impact of structural data obtained in laboratory tests on engine design goals. Design confirmation tests, prior to full-scale engine testing, are becoming more important; this is especially true in lightweight designs where state of the art analysis methods may be inadequate. Techniques, such as the one presented, are necessary to minimize the time required to evaluate the results of these designs and incorporate the results into the design system. An accurate mathematical model is required if the designer expects to be able to accurately troubleshoot his design.

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